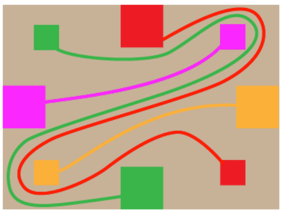
SUMMER QUIZ 2012
SOLUTIONS – PART 2

Medium 1

Find four paths that connect the squares of the same colour. The paths should not intersect and should not leave the brown playing area.

Answer: There are many possible solutions.



A very nice way to see how to solve this puzzle is to first imagine each little square very close to its large companion of the same colour; in that case it is easy to connect the squares as desired. Then just imagine the connecting paths as rubber bands, and shift the little squares back to their original positions.

Medium 2

A standard grandfather clock strikes once for 1 o'clock, twice for 2 o'clock and so on. Roman clocks have two bells, the first striking once for each I and the second striking once for each V. (Here, X counts as two V's and the number four is written as IV rather than IIII). During a twelve hour period which clock will strike more often?

Answer: The grandfather clock.

The grandfather clock will strike 1 + 2 + 3 + … + 12 = 78 times. On the Roman clock there are 17 I's, 5 V's and 4 X's, which sums to 17 + 5 + 2 x 4 = 30 strikes.

Medium 3

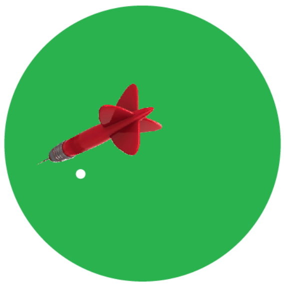
The hot water tap fills the bathtub in 9 minutes and the cold water tap fills it in 6 minutes. How long will it take to fill the tub if both taps are used?

Answer: 3 minutes and 36 seconds.

There are two ways to work this out. The sneakier method is to imagine filling lots of bathtubs. The hot tap will fill two tubs in 18 minutes and the cold tap will fill three tubs in the same time. So, together the taps will fill five tubs in 18 minutes, or one tub in 18/5 minutes.

A more direct method is to note that the *speed* of the hot tap is 1/9 tubs per minute, and the speed of the cold tap is 1/6 tubs per minute. So, the combined speed is 1/9 + 1/6 tubs per minute. That’s 5/18 tubs per minute, and so it takes 18/5 minutes for each tub.

Medium 4



Your friend tosses a 10 cent coin somewhere on the MCG. You are above in a hot air balloon and you blindly toss a dart down onto the field. What are the chances that your dart hits the coin? The diameter of the coin is about 24 millimetres, and you can assume the MCG is also circular, with a diameter of 160 metres. (The chances turn out to be about the same chance as that of a ticket winning Division 1 in Oz Lotto).

Answer: About 1 in 44 million.

The probability of hitting the coin is just the area of the coin divided by the area of the MCG. For that we just have to square the ratio of the radii, so the odds are 1 in $(160 \times 1000 / 24)^2$, which comes to 1 in 44,444,444.

Medium 5

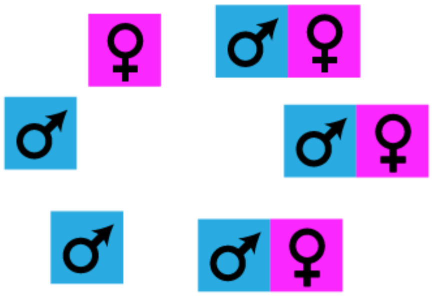
$$\frac{3}{7}+\frac{5}{6}=\frac{8}{13}$$

A popular method of adding fractions is to separately add the numerators and the denominators. Does this method ever actually give the correct answer? You can assume the numerator and denominator are both positive.

Answer: No.

The result of separately adding the numerators and denominators is known as the *mediant*. It is not too difficult to show (exercise!) that the mediant of two (positive) fractions lies between the original fractions. But of course the sum of two (positive) fractions is greater than both of the original fractions. So, the median and the sum can never be equal.

Medium 6



100 adults inhabit a village. There are 62 women and 14 single men. How many married men are there in the village?

Answer: 24.

Let M, W and C stand for the numbers of single men, single women and married couples, respectively. Then we know

$M + W + 2C = 100$

$W + C = 62$

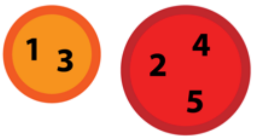
$M = 14$

The first two equations imply that

$M + C = 38.$

Then, from the third equation it follows that there are 24 couples, and so also 24 married men.

Medium 7



As pictured, the numbers 1, 2, 3, 4 and 5 can be split into two groups so that no two numbers and their sum appear in the same group. Can you do the same with the numbers 1 through 6? How far can you go?

Answer: The numbers 1 to 8 can be split into two groups.

The numbers 1 to 8 can be split into the two groups {1,2,4,8} and {3,5,6,7}, and we show below that this is the only way of splitting up these numbers. So, since 9 (= 1+ 8 = 3 + 6) cannot now be placed in either group, that proves the numbers 1 to 9 cannot be split into two groups.

To see how the numbers from 1 to 8 can be grouped, we have to consider a few cases. First, if 1 and 2 are in the same group then 3 (= 1 + 2) must be in the second group; but it's now impossible for 4 to also be in the second group:

{1, 2} {3, 4} → {1, 2, 7} {3, 4} → {1, 2, 7} {3, 4, 5, 6, 8} → no good!

So, if 1 and 2 are in the same group then we must have 4 in that group as well, and we wind up with

{1, 2, 4} {3} → {1, 2, 4} {3, 5, 6} → {1, 2, 4, 8} {3, 5, 6} → {1, 2, 4, 8} {3, 5, 6, 7}

Next, we show that it is impossible for 1 and 2 to be in different groups. There are two cases to consider:

{1, 3} {2} → {1, 3} {2, 4} → {1, 3, 6} {2, 4} → {1, 3, 6} {2, 4, 5}

→ {1, 3, 6, 7} {2, 4, 5} → no good!

{1} {2, 3} → {1, 5} {2, 3} → {1, 5} {2, 3, 4} → {1, 5, 6, 7} {2, 3, 4} → no good!

Medium 8

You want to cut a cake into identical pieces. What is the maximum possible number of pieces if you make three straight cuts?

Answer: 8

Make two vertical cuts through the centre of the cake and at right angles to each other, and then a horizontal cut along the middle of the cake.

Medium 9

$$2012^{2012}$$

What is the last digit of the monster number above?

Answer: 6.

The last digit of 2012^{2012} is the same as the last digit of 2^{2012}, and so we just have to investigate the final digits in the powers of 2. The sequence begins

$$2^1 \rightarrow 2,\ 2^2 \rightarrow 4,\ 2^3 \rightarrow 8,\ 2^4 \rightarrow 6.$$

The sequence then repeats, and so every fourth power in the sequence ends in a 6. Since 2012 is divisible by 4, the final digit in our monster number is 6.

Medium 10

You go for a swim in a circular lake. You jump in the water, swim 300 metres east and reach the edge of the lake. You then swim 400 metres south and again reach the edge of the lake again. What is the diameter of the lake?

Answer: 500 metres.

You have swum the two short sides of a right-angled triangle. By (the converse of) Thales' theorem, the long side of this triangle is a diameter of the circular lake. So, by Pythagoras's theorem this diameter is 500 metres long.